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After more than half a century the world has come to that opinion. Says F. S. Macauley in 1897, "I was simply fascinated with your translation of Bolyai's 'Science Absolute' when I first read it, as indeed almost any one would be who comes across it."

A NEW SOLUTION OF THE CUBIC EQUATION.

By DR. L. E. DICKSON.

The method here suggested is analogous to that used by Ferrari in solving the biquadratic. I multiply the reduced cubic by $x-c$ and determine c so that the resolvent cubic shall be binomial. Thus, adding $(ax+b)^2$ to each member of

$$(x-c)(x^3+px+q)=0$$

and requiring the new equation to take the form

$$[x^2-(c/2)x+\lambda+(p/6)]^2=(ax+b)^2 \tag{1}$$

we have the conditions on a, b, λ :

$$\begin{aligned} a^2&=2\lambda+\tfrac{1}{4}c^2-\tfrac{2}{3}p \\ 2ab&=-c\lambda+\tfrac{5}{6}cp-q \\ b^2&=(\lambda+\tfrac{1}{6}p)^2+qc. \end{aligned} \tag{2}$$

Since c is a root of (1), we may set

$$\tfrac{1}{2}c^2+\lambda+\tfrac{1}{6}p=ac+b, \tag{3}$$

limiting ourselves to one of the two sets of values for a and b in terms of λ and c. Equating the two values for $4a^2b^2$ given by (2), we reach the resolvent cubic for λ in *reduced* form :

$$8\lambda^3+2\lambda(3cq+c^2p-\tfrac{1}{3}p^2)+(qc^3-cpq-q^2-\tfrac{2}{3}p^2c^2-\tfrac{2}{27}p^3)=0. \tag{4}$$

Its discriminant is found to be

$$R(c^3+pc+q)^2, \text{ where } R\equiv\tfrac{1}{4}q^2+\tfrac{1}{27}p^3$$

is the discriminant of the given cubic.

The coefficient of 2λ in (4) vanishes for

$$\tfrac{1}{3}pc=-\tfrac{1}{2}q+\sqrt{R}, \tag{5}$$

whence

$$c^3+pc+q=\frac{108\,R}{p^3}(-\tfrac{1}{2}q+\sqrt{R}).$$

Hence, if we determine c by (5) the cubic (4) becomes

$$8\lambda^3=-q(c^3+pc+q)+8R=\left(\frac{6\sqrt{R}}{p}\right)^3(-\tfrac{1}{2}q+\sqrt{R}). \tag{6}$$

Then by (2), $a^2=\frac{6\sqrt{R}}{p}(-\frac{1}{2}q+\sqrt{R})+\frac{1}{4}c^2-\frac{2}{3}r$.

By a slight calculation, we find

$$a=\tfrac{1}{2}c+\sqrt[3]{-\tfrac{1}{2}q+\sqrt{R}}+\sqrt[3]{-\tfrac{1}{2}q-\sqrt{R}}.$$

This would be expected, since by (3)

$$(x-c)(x-a+\tfrac{1}{2}c)=x^2-\tfrac{1}{2}cx+\lambda+\tfrac{1}{6}p-ax-b,$$

whence $a-\frac{1}{2}c$ is a root of the given cubic.

University of California, January 17.

THE FIRST AWARD OF THE LOBACHEVSKI PRIZE.

By GEORGE BRUCE HALSTED, A. M. (Princeton); Ph. D. (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

The Lobachévski prize is adjudged every three years. Its value is five hundred roubles. It is given for work in geometry, preferably non-Euclidean geometry. All works published within the six years preceding the award of the prize and sent by their authors to the Physico-Mathematical Society of Kazan are allowed to compete if published in Russian, French, German, English, Italian, or Latin.

The Society has now in formal session awarded the prize to Sophus Lie, Professor of Mathematics at the University of Leipzig, for his work "Theorie der Transformationsgruppen. Band III. Leipzig, 1893." In this work the theory of non-Euclidean geometry has been exhaustively re-stated and re-established in a profound investigation of the work of Helmholtz on the space-problem.

To the genius of Helmholtz is due the conception of studying the essential characteristics of a space by a consideration of the movements possible therein.

But since the time when Helmholtz did his work on this subject, the greatest of living mathematicians, Sophus Lie, formerly of Christiana, has enriched mathematics with a new instrument, the Theory of Groups, which its creator has applied with tremendous power to the Hemholtz treatment. Lie finds, as was almost inevitable, that certain details had escaped the great physicist, but that, with the tact of true genius, he had kept his main results free from error, though there comes to light a superfluity in his explicit assumptions, an unconscious assumption now seen to be mathematically important for the rigor of the demonstration, and at least one definits error in minor results.

Lie's method is in general the following. Consider a tridimensional space, in which a point is defined by three quantities, x, y, z.